Pre-Calculus
9-3 Other Identities

Name Key!

Simplify (hint: you have some identities that can help you)

Relates to sum & difference identities

Sin (x + x) $= \sin x \cdot \cos x + \cos x \sin x$
$= 2\sin x \cos x$

Cos (x + x) $= \cos x \cdot \cos x - \sin x \cdot \sin x$
$= \cos^2 x - \sin^2 x$

Tan (x + x) → use original Identity
$\frac{\tan x + \tan x}{1 - \tan x \tan x} = \frac{2\tan x}{1 - \tan^2 x}$

Congratulations! You have just derived the *double angle identities* ☺

There are 2 other ways that cos(2x) can be written. Give the two other forms of cos(2x).
Hint: Use your Pythagorean identities.

Cos(2x) = $2\cos^2 x - 1$
$\cos^2 x - (1 - \cos^2 x)$
$\cos^2 x - 1 + \cos^2 x$
$2\cos^2 x - 1$

cos(2x)= $1 - 2\sin^2 x$
$(1 - \sin^2 x) - \sin^2 x$
$1 - 2\sin^2 x$

Take these two double angle identities above and solve the correct equation for $\sin^2 x$ and the correct equation for $\cos^2 x$.

$\sin^2 x = \frac{1 - \cos 2x}{2}$
$\frac{-2\sin^2 x}{-2} = \frac{\cos 2x - 1}{-2}$
$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{\cos 2x - 1}{2}$
$2\cos^2 x - 1 = \cos 2x$
$\frac{2\cos^2 x}{2} = \frac{\cos 2x - 1}{2}$
$\cos^2 x = \frac{\cos 2x - 1}{2}$

The two equations you just found are the ***Power Reducing Identities***

Now we are going to use the power reducing identities to find the *half angle identities*

Start by rewriting your power reducing identities.

$\sin^2 x = \frac{1-\cos 2x}{2}$ $\qquad\qquad$ $\cos^2 x = \frac{\cos 2x+1}{2}$

If we are looking for half angle identities, we need to replace the angle we have with **half** the angle. Do that and then solve each one (you should end up with two equations, one that says sin x = and one that says cos x =)

$\sin\frac{x}{2} =$

$$\sqrt{\sin^2\frac{x}{2}} = \frac{1-\cos\frac{2x}{2}}{2} = \sqrt{\frac{1-\cos x}{2}}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$\cos\frac{x}{2} =$

$$\cos^2\frac{x}{2} = \frac{\cos\frac{2x}{2}-1}{2}$$

$$\sqrt{\cos^2\frac{x}{2}} = \sqrt{\frac{\cos x-1}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{\cos x+1}{2}}$$

Now that you know $\sin\frac{x}{2}$ and $\cos\frac{x}{2}$, find the identity for $\tan\frac{x}{2}$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\pm\sqrt{\frac{1-\cos x}{\sqrt{2}}}}{\pm\sqrt{\frac{\cos x-1}{\sqrt{2}}}} = \pm\frac{\sqrt{1-\cos x}}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{\sqrt{\cos x-1}}$$

$$= \pm\frac{\sqrt{1-\cos x}}{\sqrt{\cos x-1}}$$

These are your *Half-Angle Identities.*

<u>Practice with these identities:</u>

<u>Directions:</u> Use the half angle identities to evaluate the given expression exactly.

1. $\cos\frac{\pi}{8}$

$$\frac{\frac{\pi}{8}}{2} = \frac{\pi}{4}$$

$$= \pm\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$

$$= \pm\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$$

2. $\sin\frac{5\pi}{8}$

$$x = \frac{5\pi}{4}$$

$$\sin\frac{5\pi}{8} = \pm\sqrt{\frac{1-\cos\frac{5\pi}{4}}{2}}$$

$$= \pm\sqrt{\frac{1+\left(+\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= \pm\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \pm\frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}}$$

$$= \pm\frac{\sqrt{2+\sqrt{2}}}{4}$$

3. $\cot\frac{\pi}{8} = \frac{1}{\tan\frac{\pi}{8}}$

$$x = \frac{\pi}{4}$$

$$= \frac{\sin\frac{\pi}{4}}{1-\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}$$

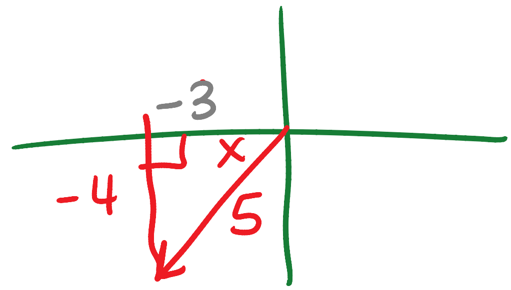
$$= \frac{\frac{\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}}$$

$$= \frac{\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}$$

$$= \frac{2\sqrt{2}+2}{4-2=2} = \sqrt{2}+1$$

<u>Directions:</u> Find sin2x, cos2x, and tan2x under the given conditions.

4. $\sin x = -\frac{4}{5}$ for $\pi < x < \frac{3\pi}{2}$ 3rd quad



$$\sin 2x = 2\sin x\cos x$$
$$= 2(-4/5)(-3/5)$$
$$= 24/5$$

$$(-4)^2 + x^2 = 5^2$$
$$x^2 = 25 - 16 = 9$$
$x = 3 \rightarrow$ 3rd quad so (-)

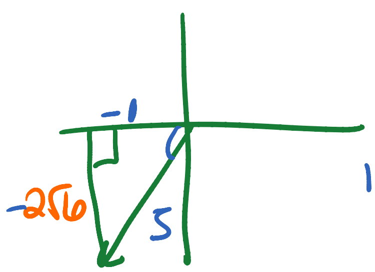
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= (-3/5)^2 - (-4/5)^2$$
$$= 9/25 - 16/25 = -7/25$$

$$\tan 2x = \frac{24/5}{-7/25} = -24/7$$

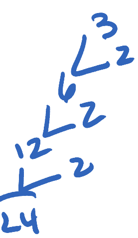
5. $\sec x = -5$ for $\pi < x < \frac{3\pi}{2}$

$\cos x = \frac{A}{H}$

$\sec x = \frac{H}{A}$



$$1^2 + x^2 = (-5)^2$$
$$x^2 = 25 - 1$$
$$x^2 = 24 \quad x = \sqrt{24}$$
$$x = 2\sqrt{6}$$



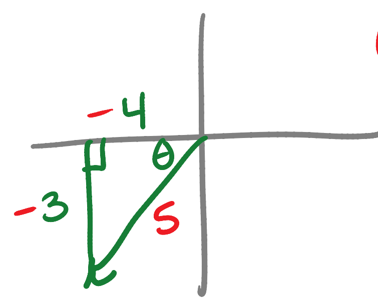
$$\cos 2x = (-1/5)^2 - \left(\frac{-2\sqrt{6}}{5}\right)^2$$
$$= 1/25 - \frac{4 \cdot 6}{25} = \frac{1-24}{25} = \frac{-23}{25}$$

$$\sin 2x = 2\left(\frac{-2\sqrt{6}}{5}\right)(-1/5) = \frac{4\sqrt{6}}{25}$$

$$\tan 2x = \frac{2(2\sqrt{6})}{1-(2\sqrt{6})^2} = \frac{4\sqrt{6}}{1-4\cdot 6} = \frac{4\sqrt{6}}{23}$$

6. $\tan x = \frac{3}{4}$ for $\pi < x < \frac{3\pi}{2}$

$$(-3)^2 + (-4)^2 = x^2$$
$$9 + 16 = x^2$$
$$x = 5$$



$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = (-4/5)^2 - (3/5)^2$$
$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

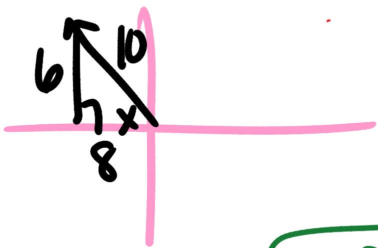
$$\sin 2x = 2(-4/5)(-3/5)$$
$$= 24/5$$

$$\tan 2x = \frac{2(3/4)}{1-(3/4)^2} = \frac{6/4}{1-9/16}$$
$$= \frac{3/2}{7/16} = \frac{24}{7}$$

Directions: Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, and $\tan\frac{x}{2}$ under the given conditions.

7. $\sin x = .6$ for $\frac{\pi}{2} < x < \pi$ 2nd quad

$.6 = \frac{6}{10}$



$6^2 + x^2 = 10^2$

$x^2 = 100 - 36$

$x = \sqrt{64}$

$x = 8$

$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - 8/10}{2}}$

$= \pm\sqrt{\frac{1}{10}}$

$= \pm\frac{1}{\sqrt{10}} = +\frac{\sqrt{10}}{10}$

in 2nd quad so (+)

$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + 8/10}{2}}$

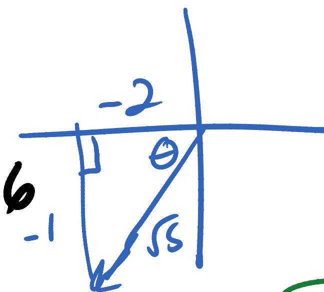
$= \pm\sqrt{9/10} = \pm\frac{3}{\sqrt{10}} = \pm\frac{3\sqrt{10}}{10}$

$\tan\frac{x}{2} = \frac{1 - 8/10}{6/10} = \frac{1/5}{6/10} = 1/3$

8. $\tan x = \frac{1}{2}$ for $\pi < x < \frac{3\pi}{2}$

$1^2 + 2^2 = x^2$

$\sqrt{5} = x$



$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \frac{-2}{\sqrt{5}}}{2}}$

$= \pm\sqrt{\frac{\sqrt{5}+2}{2\sqrt{5}}}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + -2/\sqrt{5}}{2}}$

$= \pm\sqrt{\frac{\sqrt{5}-2}{2}}$

$\tan\frac{x}{2} = \frac{\frac{-1}{\sqrt{5}}}{1 + \frac{-2}{\sqrt{5}}} = \frac{\frac{-1}{\sqrt{5}}}{\frac{\sqrt{5}-2}{\sqrt{5}}}$

$= \frac{-1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}-2}$

$= \frac{-1}{\sqrt{5}-2} \cdot \frac{(\sqrt{5}+2)}{\sqrt{5}+2}$

$= \frac{-\sqrt{5}-2}{5-4} = -\sqrt{5}-2$

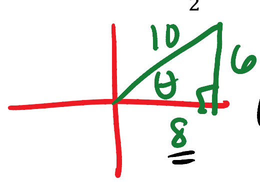
Directions: Assume $\sin x = .6$ for $0 < x < \frac{\pi}{2}$, and evaluate the given expression.

9. $\sin 4x$

$\sin(2x + 2x) =$

$2\sin(2x)\cos(2x)$

$2\left(\frac{18}{25}\right)\left(\frac{7}{25}\right) = \frac{252}{625}$



$6^2 + x^2 = 10^2$

$\sin 2x = 2\left(\frac{6}{10}\right)\left(\frac{6}{10}\right)$

$= \frac{72}{100} = \frac{18}{25}$

10. $\cos\frac{x}{2} = \pm\sqrt{\frac{1 + 8/10}{2}}$

$= \pm\sqrt{\frac{18/10}{2}}$

$= \pm\sqrt{9/10} = +\frac{3}{\sqrt{10}}$

.9487

1st quad

$\cos 2x = (8/10)^2 - (6/10)^2$

$= \frac{64}{100} - \frac{36}{100} = \frac{28}{100} = 7/25$

$\frac{252}{625} \approx .4032$

Directions: Simplify the given expression.

11. $\frac{sin2x}{2sinx}$

$\frac{2\cancel{\sin x}\cos x}{\cancel{2\sin x}}$

$= \cos x$

12. $2cos2ysin2y$

$\sin(2x) = 2\cos y \sin y$

$\sin(2x) = 2\cos y \sin y$

$\sin(2x+2x) = 2\cos 2y \sin 2y$

$\sin(4x)$

13. $(sinx + cosx)^2 - sin2x$

$\sin^2 x + 2\sin x\cos x + \cos^2 x - \sin 2x$

$1 + 2\sin x\cos x - \sin 2x$

$1 + \sin 2x - \sin 2x$

$= \boxed{1}$